

Collectivity Embedded in Complex Spectra: Example of Nuclear Double-Charge Exchange Modes

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The mechanism of collectivity coexisting with chaos is investigated on the quantum level. The complex spectra are represented in the basis of two-particle two-hole states describing the nuclear double-charge exchange modes in ^{48}Ca . An example of $J^\pi = 0^-$ excitations shows that the residual interaction, which generically implies chaotic behavior, under certain specific and well identified conditions may create transitions stronger than those corresponding to the pure mean-field picture. Therefore, for this type of excitations such an effect is not generic and in most cases the strength of transitions is likely to take much lower values, even close to the Porter-Thomas distributed.

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Chaos is essentially a generic property of complex systems such as atomic nuclei and this finds evidence in a broad applicability of the random matrix theory (RMT) [1] to describe level fluctuations. Another characteristics connected with complexity, even more interesting and important from the practical point of view, is collectivity. It means a cooperation, and thus the coupling, between the different degrees of freedom in order to generate a coherent signal in response to an external perturbation. Consequently, even though the real collectivity implies a highly ordered behavior it involves effects beyond the mean field – the most regular part of the nuclear Hamiltonian [2]. At the same time the effects beyond the mean field are responsible for the fluctuation properties characteristic of the Gaussian orthogonal ensemble (GOE) of random matrices [3].

Local level fluctuations characteristic of GOE appear [4] to take place for the nuclear Hamiltonian acting already in the space of two-particle – two-hole (2p2h) states and this is a crucial element for an appropriate description of the giant resonance decay properties [5]. The ordinary giant resonances are, however, excited by one-body operators which directly probe the 1p1h components of the nuclear wave function. The 2p2h states only form the background which determines a decay-law. There exist, however, very interesting physical processes, represented by two-body external operators, which directly couple the ground state to the space of 2p2h states. In view of the above mentioned local GOE fluctuations giving evidence for a significant amount of chaotic dynamics already in the 2p2h space, the question of a pos-

sible coherent response under such conditions is a very intriguing one and of interest not only for many branches of physics but also for biological systems [6].

Among various nuclear excitation modes which can be considered in this context the double charge exchange (DCX) processes are of special interest. These modes, excited in (π^+, π^-) reactions [7], involve at least two nucleons within the nucleus and give rise to a sharp peak at around 50 MeV in the forward cross section. They are thus located in the energy region of the high density of 2p2h states which points to the importance of coherence effects among those states. Consequently, the present investigation may also appear helpful in future studies of the mechanism of DCX reactions and in separating the suggested [8] dibaryon contribution from the conventional effects [9].

Diagonalizing the nuclear Hamiltonian in the subspace of 2p2h states $|2\rangle \equiv a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1} |0\rangle$ yields the eigenenergies E_n and the corresponding eigenvectors $|n\rangle = \Sigma_2 c_2^n |2\rangle$. In response to an external field \hat{F}_α a state $|F_\alpha\rangle \equiv \hat{F}_\alpha |0\rangle = \sum_n \langle n | \hat{F}_\alpha | 0 \rangle |n\rangle$ is excited. The two-phonon operator \hat{F}_α can be represented as $\hat{F}_\alpha = \{\hat{f}_\beta \otimes \hat{f}_\gamma\}_\alpha$, where \hat{f}_β and \hat{f}_γ denote the single-phonon operators whose quantum numbers β and γ are coupled to form α . The state $|F_\alpha\rangle$ determines the strength function

$$S_{F_\alpha}(E) = \sum_n S_{F_\alpha}(n) \delta(E - E_n), \quad (1)$$

where $S_{F_\alpha}(n) = |\langle n | \hat{F}_\alpha | 0 \rangle|^2$. In the basis of states $|2\rangle$

$$S_{F_\alpha}(n) = \sum_2 |c_2^n|^2 |\langle 2 | \hat{F}_\alpha | 0 \rangle|^2 + \sum_{2 \neq 2'} c_2^{n*} c_2^n \langle 0 | \hat{F}_\alpha^\dagger | 2' \rangle \langle 2 | \hat{F}_\alpha | 0 \rangle. \quad (2)$$

This equality defines its diagonal ($S_{F_\alpha}^d(n)$) and off-diagonal ($S_{F_\alpha}^{od}(n)$) contributions. The second component includes many more terms and it is this component which potentially is able to induce collectivity, i.e. a strong transition to energy E_n . Two elements are however required: (i) a state $|n\rangle$ must involve sufficiently many expansion coefficients c_2^n over the unperturbed states $|2\rangle$ which carry the strength ($\langle 2 | \hat{F}_\alpha | 0 \rangle \neq 0$) and this is equivalent to at least local mixing, but at the same time (ii) sign correlations among these expansion coefficients

should take place so that the different terms do not cancel out.

Optimal circumstances for the second condition to be fulfilled read: $c_2^n \sim \langle 0 | \hat{F}_\alpha | 2 \rangle$. This may occur if the interaction matrix elements can be represented by a sum of separable terms \mathbf{Q}^ν of the multipole-multipole type ($\mathcal{V}_{ij,kl} = \sum_{\nu=1}^M Q_{ij}^\nu Q_{kl}^\nu$ with $Q_{ij}^\nu \sim \langle i | \hat{f}_\nu | j \rangle$). The success of the Brown-Bolsterli schematic model [10] in indicating the mechanism of collectivity on the 1p1h level points to an approximate validity of such a representation. Collectivity can then be viewed as an edge effect connected with appearance of a dominating component in the Hamiltonian matrix and the rank M of this component is significantly lower (unity in case of the Brown-Bolsterli model) than the size of the matrix. This rank specifies a number of the prevailing states whose expansion coefficients predominantly are functions of \mathbf{Q}^ν . Due to two-body nature of the nuclear interaction which reduces its 2p2h matrix elements to combinations of the ones representing the particle-particle, hole-hole and particle-hole interactions [4] the separability may become effective also on the 2p2h level.

For quantitative discussion presented below we choose the ^{48}Ca nucleus, specify the mean field part of the Hamiltonian in terms of a local Woods-Saxon potential including the Coulomb interaction and adopt the density-dependent zero-range interaction of ref. [11] as a residual interaction. Since we want to inspect the higher energy region at least three mean field shells on both sides of the Fermi surface have to be used to generate the unperturbed 2p2h states as a basis for diagonalization of the full Hamiltonian. Typically, the number of such states is very large and this kind of calculation can be kept under full numerical control only for excitations of the lowest multipolarity. For this reason we perform a systematic study of the DCX $J^\pi = 0^-$ states. Our model space then develops $N=2286$ 2p2h states. There are still several possibilities of exciting such a double-phonon mode represented by the operator \hat{F}_α out of the two single-phonons \hat{f}_β and \hat{f}_γ of opposite parity. For definiteness we choose $\hat{f}_\beta = rY_1\tau_-$ and $\hat{f}_\gamma = r^2[Y_2 \otimes \sigma]^{1+}\tau_-$. The first of these operators corresponds to the $1\hbar\omega$ dipole and the second to $2\hbar\omega$ spin-quadrupole excitation. The resulting two-phonon mode thus operates on a level of $3\hbar\omega$ excitations.

The results of calculations are presented in Fig. 1. As one can see, including the residual interaction (part (b)) induces a spectacularly strong transition at 49.1 MeV. This transition is stronger by more than a factor of 2 than any of the unperturbed (part (a)) transitions even though it is shifted to significantly higher (~ 10 MeV) energy. This is also a very collective transition. About 96% of the corresponding strength originates from $S_{F_\alpha}^{od}(n)$, as comparison between parts (b) and (c) of Fig. 1 documents. The degree of mixing can be quantified, for instance, in terms of the information entropy [12] $I(n) = -\sum_i p_i \ln p_i$ ($p_i = |c_i^n|^2$) of an eigenvector $|n\rangle$ in the basis (part (d)).

Interestingly, the system finds preferential conditions for creating the most collective state in the energy region of local minimum in $I(n)$. Our following discussion is supposed to shed more light on this issue.

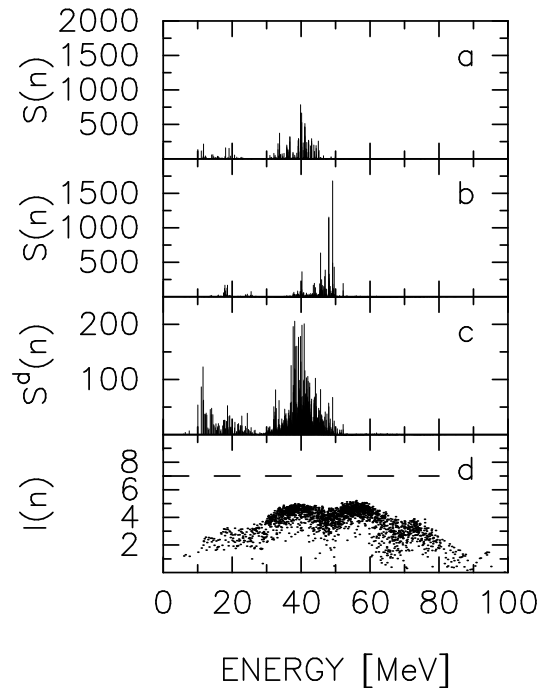


FIG. 1. (a) The unperturbed transition-strength distribution in ^{48}Ca for the $J^\pi = 0^-$ DCX excitation involving the single-phonon dipole and $2\hbar\omega$ spin-quadrupole modes. (b) The same as (a) but after including the residual interaction. (c) $S_{F_\alpha}^{od}(n)$ components of the transition-strength as defined by eq. (2) (notice a different scale). (d) The information entropy of the states $|n\rangle$ in the unperturbed basis. The dashed line indicates the GOE limit ($\ln(0.48N)$).

As shown in Fig. 2(a) our Hamiltonian matrix displays a band-like structure with spots of the significant matrix elements inside. This together with the nonuniform energy distribution $\rho_u(E)$ of the unperturbed 2p2h states, (Fig. 2(b)) which is a trace of the shell structure of the single particle states, sizably suppresses the range of mixing and locally supports conditions for the edge effect to occur in the energy region of the minimum in $\rho_u(E)$. A comparison with Fig. 1(b) shows that the collective state is located at about this region. Moreover, the minimum survives diagonalization ($\rho_p(E)$ in Fig. 2(c)) and all the above features are consistent with the effective band range [13] $((\Delta E_i)^2 = \sum_j (H_{ii} - H_{jj})^2 H_{ij}^2 / \sum_{i \neq j} H_{ij}^2)$ shown in Fig. 2(d).

Further quantification of the character of mixing between the unperturbed states is documented in Fig. 3. The distribution of off-diagonal matrix elements (a) is of the Porter-Thomas (P-T) – type which indicates the presence of the dominating multipole-multipole components in the interaction [3,14].

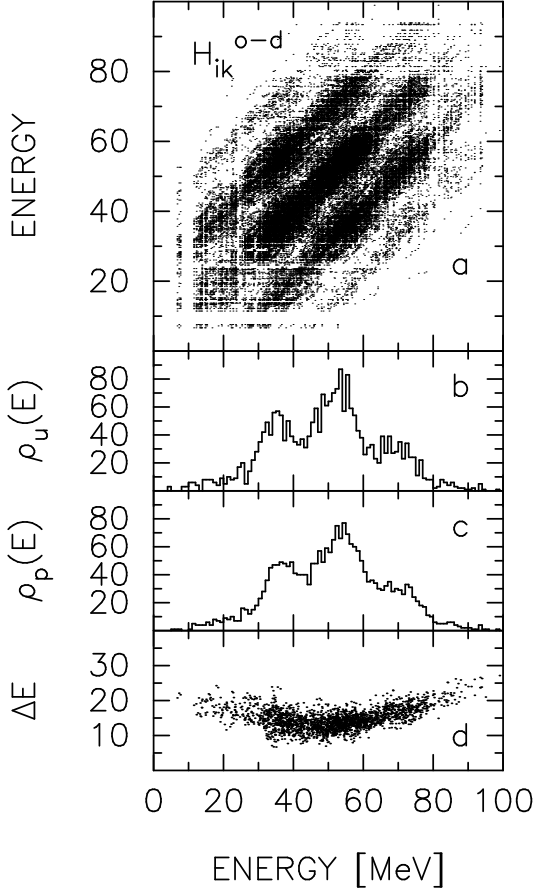


FIG. 2. (a) The structure of the Hamiltonian matrix for the $J^\pi = 0^-$ DCX states. The states are here labeled by energies, ordered in ascending order and the matrix elements $H_{ik} \geq 0.1$ are indicated by the dots. (b) The density of the unperturbed 2p2h-states. (c) The density of states after the diagonalization. (d) The energy range of interaction between the unperturbed states.

An interesting novel feature is the asymmetry between the positive and negative valued matrix elements (see parameters in the caption to Fig. 3). The positive matrix elements are more abundant which expresses further correlations among them and the fact that the interaction is predominantly repulsive for the mode considered. As a chaos related characteristics we take the spectral rigidity measured in terms of the Δ_3 statistics [1]. We find this measure more appropriate for studying various local subtleties of mixing than the nearest neighbor spacing (NNS) distribution because for a smaller number of states the latter sooner becomes contaminated by strong fluctuations. Indeed, the spectral rigidity (Fig. 3(b)) detects differences in the level repulsion inside the string of eigenvalues (i1) covering the first maximum in $\rho_{p(u)}(E)$ (35.0-41.75 MeV) and the one (i2) covering the minimum and thus including the collective state (41.75-50.1 MeV). Again, the deviation from GOE is more significant in i2

which, similarly as the behavior of $I(n)$, signals a more regular dynamics in the vicinity of the collective state ($n = 996$).

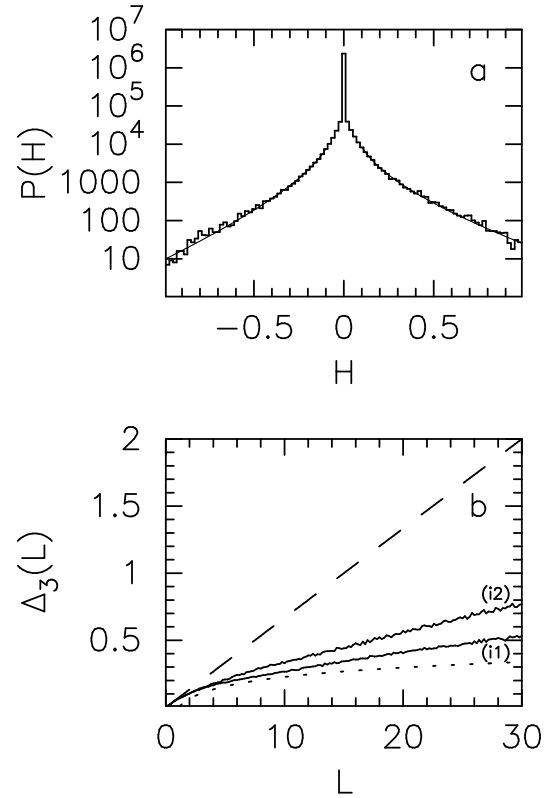


FIG. 3. (a) The distribution of off-diagonal matrix elements between the $J^\pi = 0^-$ DCX states (histogram). The solid lines indicate fit in terms of $P(H) = a|H|^b \exp(-|H|/c)$ with the resulting parameters: $a = 683$, $b = -1.22$, $c = 0.23$ (left) and $a = 538$, $b = -1.32$, $c = 0.32$ (right). (b) The spectral rigidity $\Delta_3(L)$ for eigenvalues from the two intervals: $n = 351 - 700$ (i1) and $n = 701 - 1050$ (i2). The long-dashed line corresponds to Poisson level distribution and the short-dashed line to GOE.

The conditions corresponding to the actual nuclear Hamiltonian are not the most optimal ones from the point of view of the collectivity of our $J^\pi = 0^-$ DCX excitation. By multiplying the off-diagonal matrix elements by a factor of 0.7 we obtain a picture as shown in Fig. 4(a and b). Now the transition located at 46 MeV is another factor of 2 stronger than before. However, the range of values of a multiplication factor which produces this kind of picture is rather narrow and this feature of collectivity resembles a classical phenomenon of the stochastic resonance [15]. It is relatively easy to completely destroy such a strong transition. By multiplying the off-diagonal matrix elements by a factor of 3 (which is equivalent to increasing the density of states) the strength distribution displays a form as shown in Fig. 4(c).

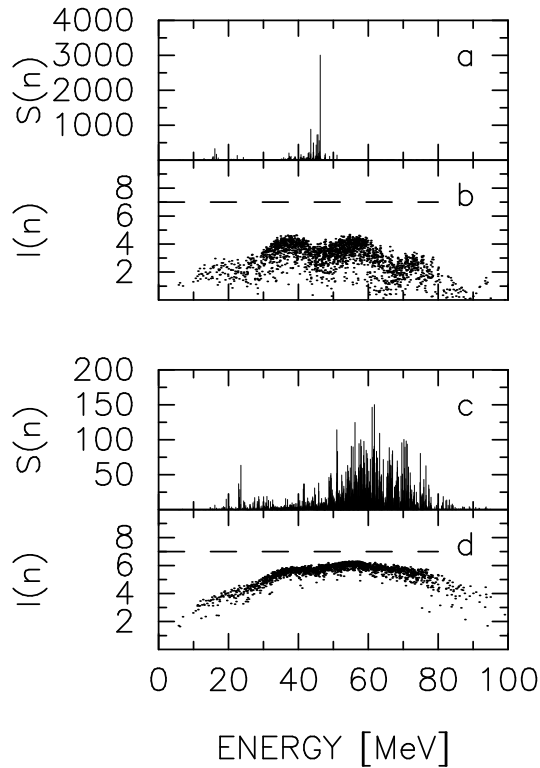


FIG. 4. The transition-strength distribution and the information entropy for the same excitation as in Fig. 1 but off-diagonal matrix elements are now multiplied by a factor of 0.7 ((a) and (b)) and by a factor of 3.0 ((c) and (d)).

Even though this strength remains largely localized in energy (standard way of analysing experimental data may even classify it as collective) the corresponding $P(S(n))$ is essentially P-T distributed (Gaussian distributed amplitudes) which becomes evident from the entropy estimate. Normalizing the total strength to unity, identifying the so normalized $S(n)$ with the probability ρ_n to populate a state at energy E_n , and defining the corresponding total entropy $\mathcal{I} = -\sum_n \rho_n \ln \rho_n$ one obtains $\mathcal{I} = 6.82$ while the P-T distribution for the same number of states ($N = 2286$) gives $\mathcal{I} = 7.0$. Further increase of the multiplication factor may again produce some transitions which are more collective than those allowed by P-T. In particular, starting from values ~ 5 some new strong collective transitions appear at the upper edge of the whole spectrum but this is the effect of basis truncation.

In conclusion, a real collectivity, by which we mean a transition stronger than those generated by the mean field, is a very subtle effect and is not a generic property of the complex spectra. Its appearance, as it happens for one of the components of the $J^\pi = 0^-$ DCX excitations considered here, involves several elements like correlations among the matrix elements, nonuniformities in the distribution of states and a proper matching of the interaction strength to an initial (unperturbed) lo-

cation of the transition strength relative to the scale of nonuniformities in the distribution of states. If present, a collective state is then located in the region of more regular dynamics. This aspect of collectivity parallels an analogous property hypothesised for living organisms [6] and stating that collectivity is a phenomenon occurring at the border between chaos and regularity. Based on the present study it is expected that majority of the two-phonon nuclear giant resonances is characterised by a spectrum of transitions which are significantly smaller than those corresponding to the mean field picture and may even be compatible with the P-T distribution with the largest transitions concentrated at similar energies. This latter effect may then lead to an illusion of collectivity.

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